

# Optimizing minor machines for fame and glory

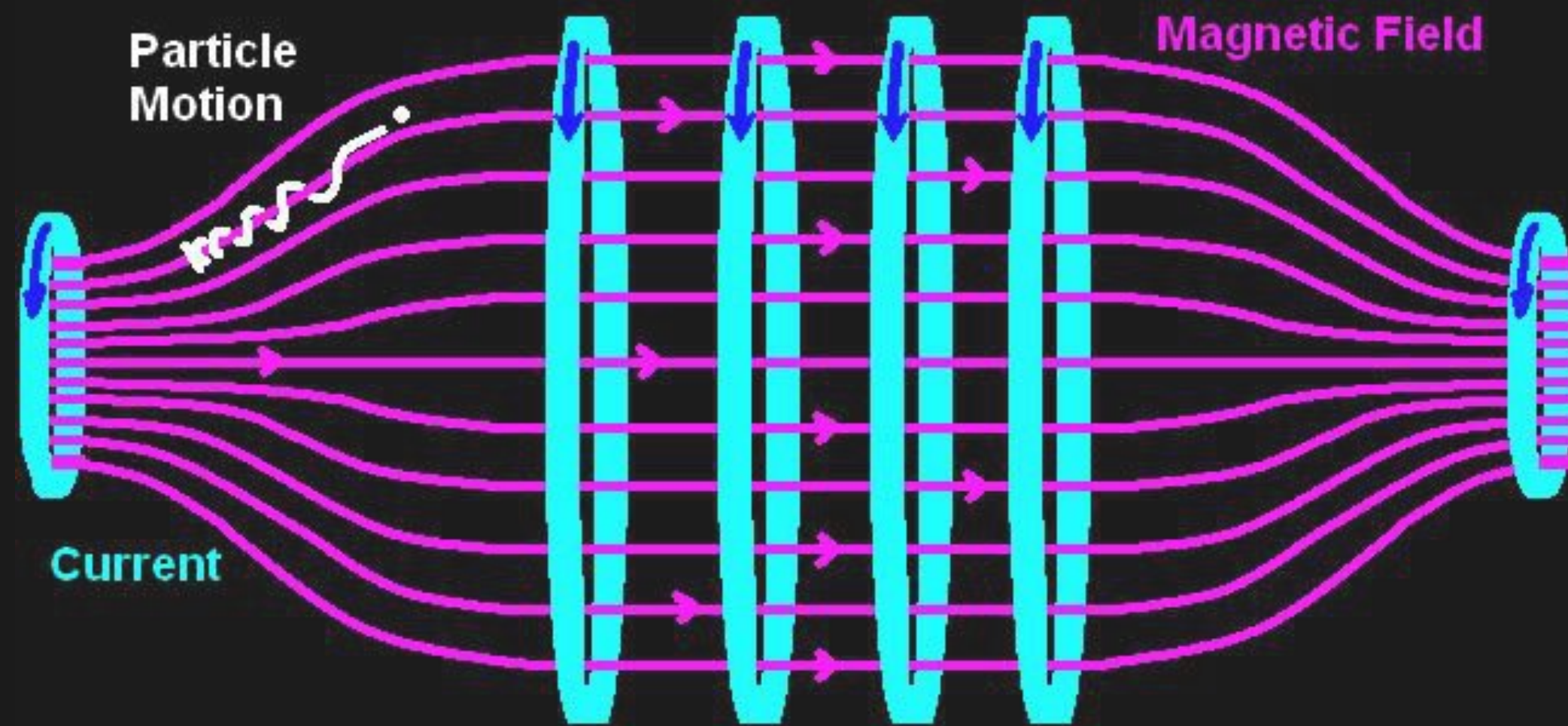
Phil Travis

APS DPP 2023 student dayyyy lmao

**(Kunal technically helped with this project but any errors / emotional suffering caused by this talk is solely my own doing)**

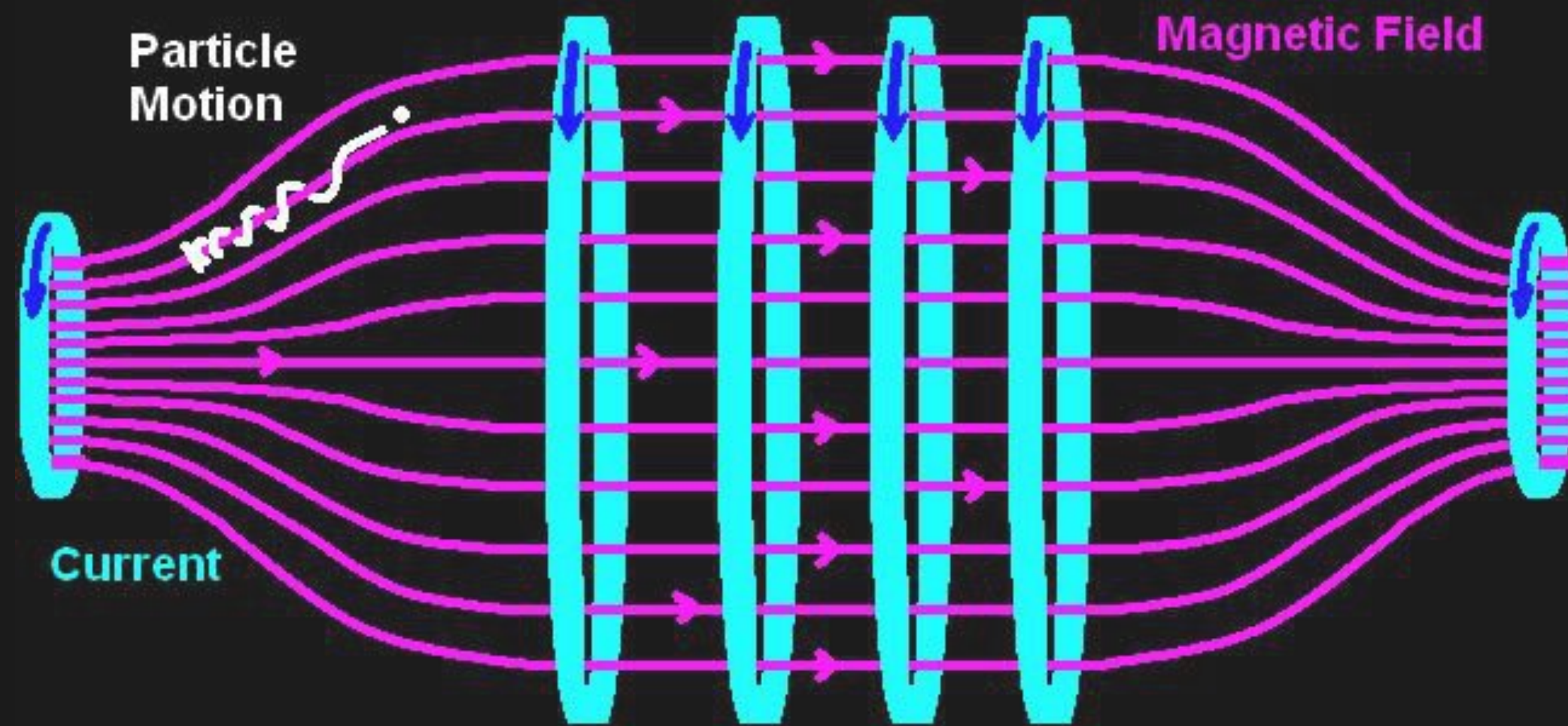
Mirror machines: old fusion concept, basis is conservation of  $\mu$

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Dr . Moynihan

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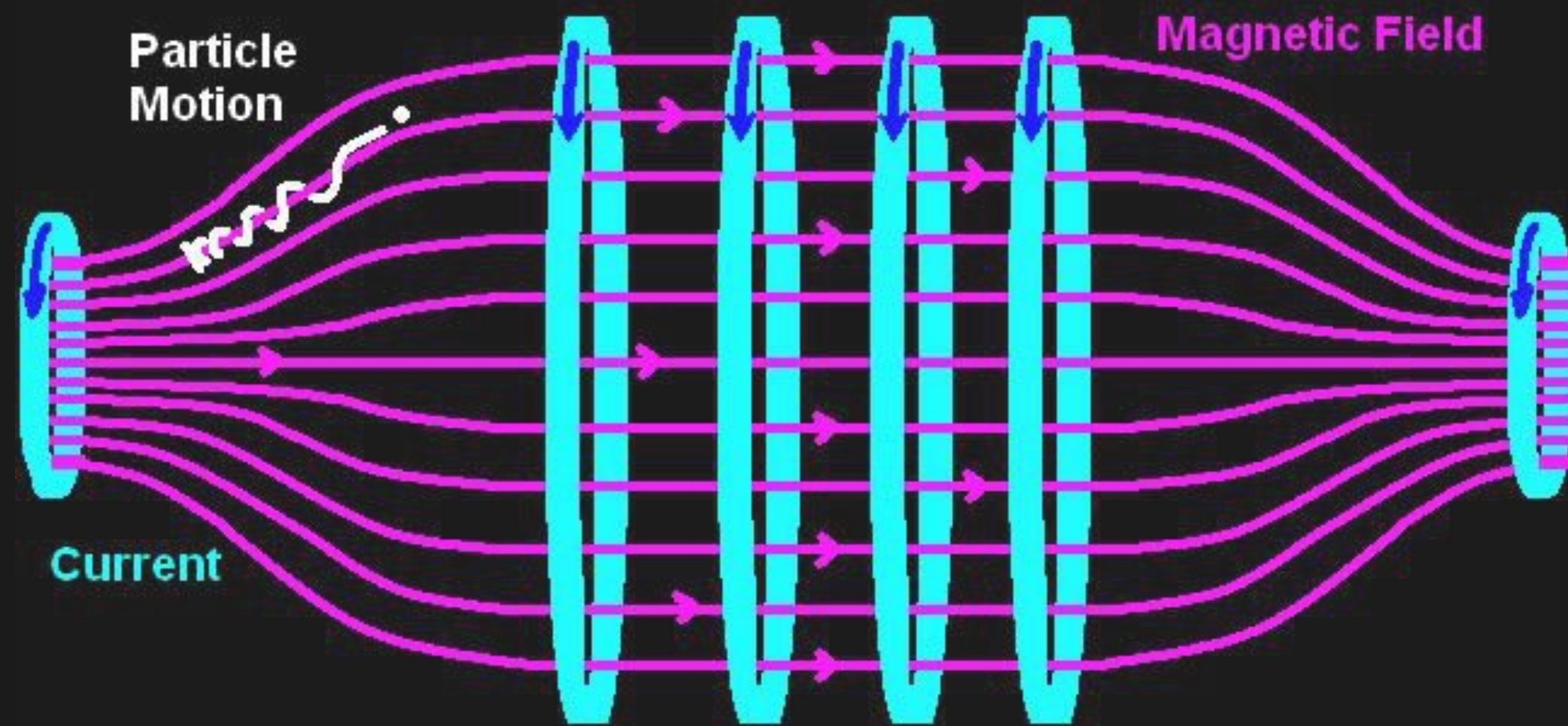


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Conservation of magnetic moment:

$$\mu = \frac{W_{\perp}}{B}$$

# Mirror machines: old fusion concept, basis is conservation of $\mu$

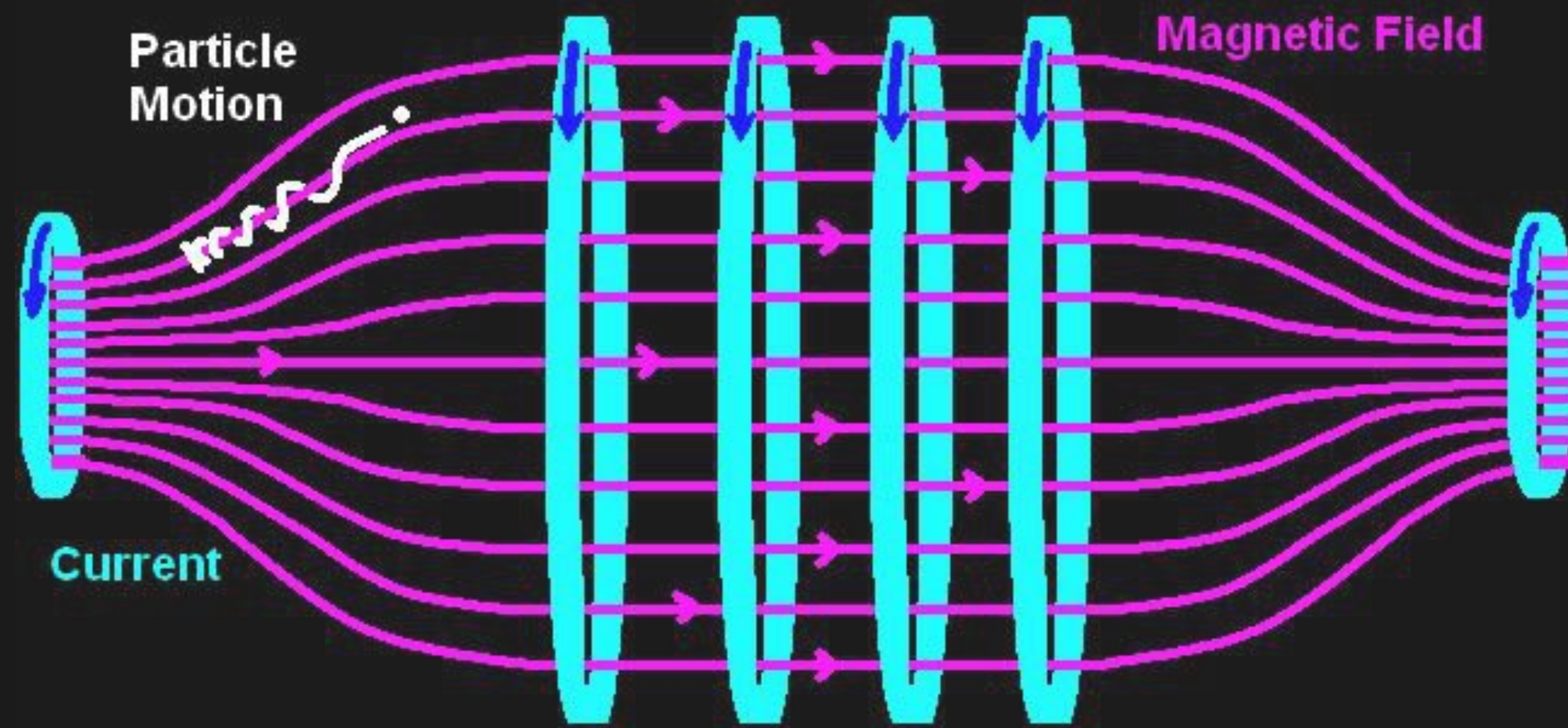


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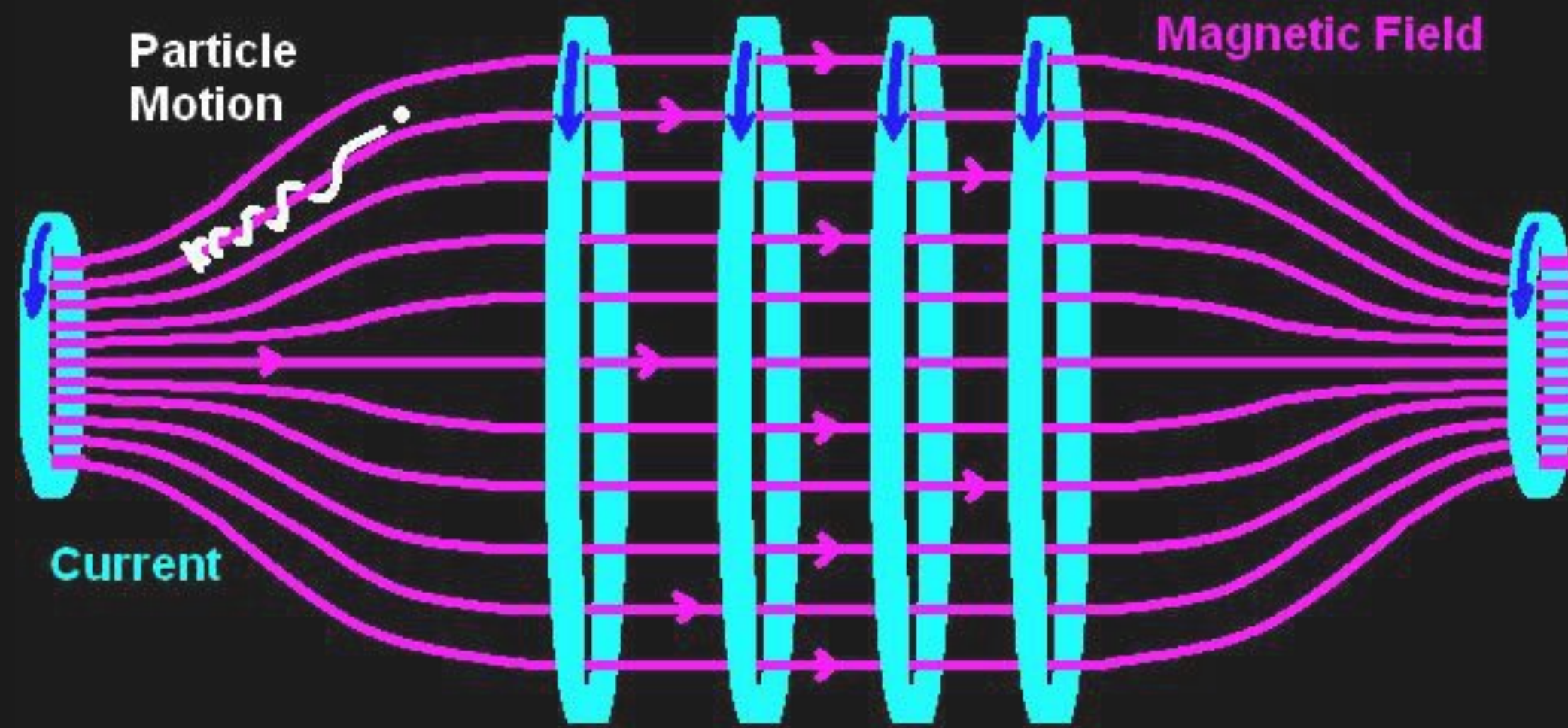


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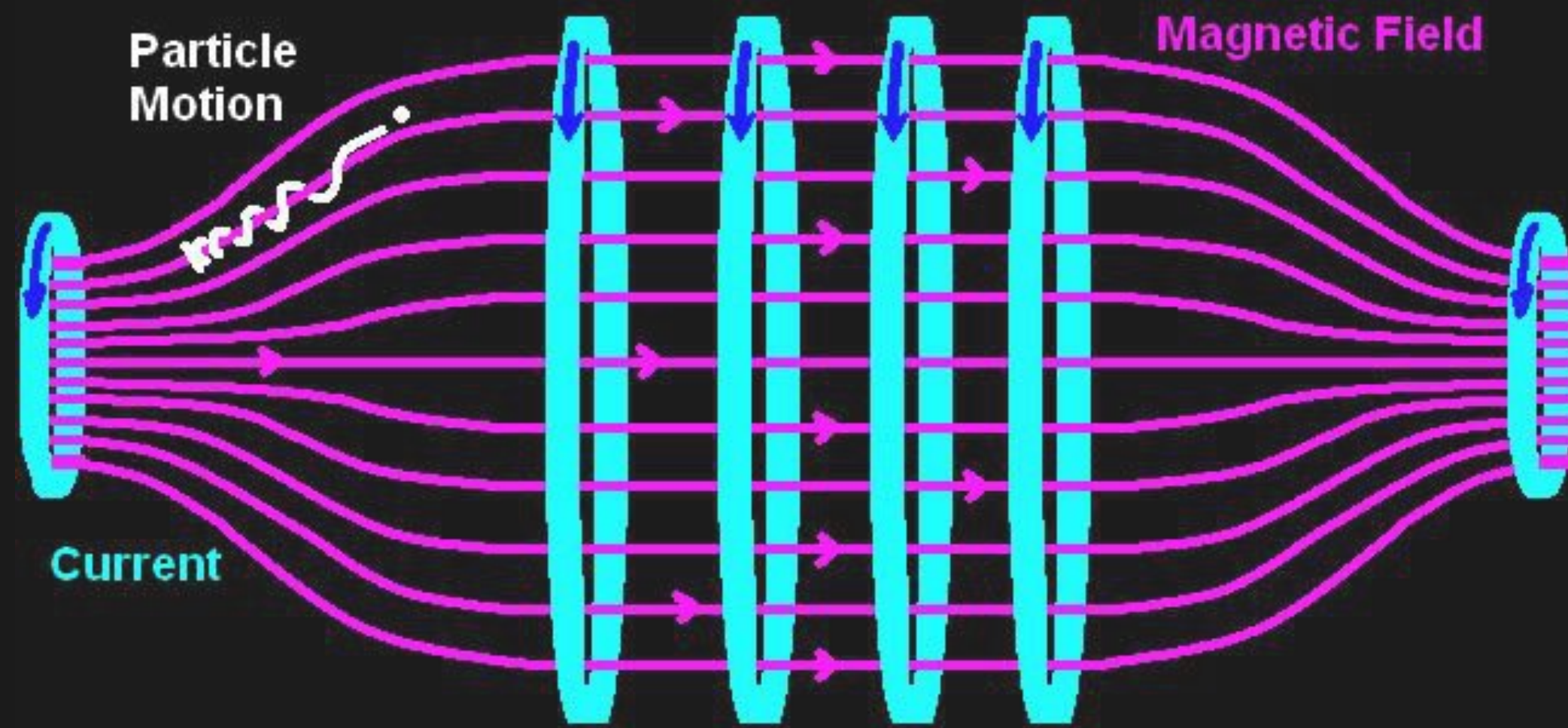
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Conservation of magnetic moment:

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conservation of  
energy

# Mirror machines: old fusion concept, basis is conservation of $\mu$



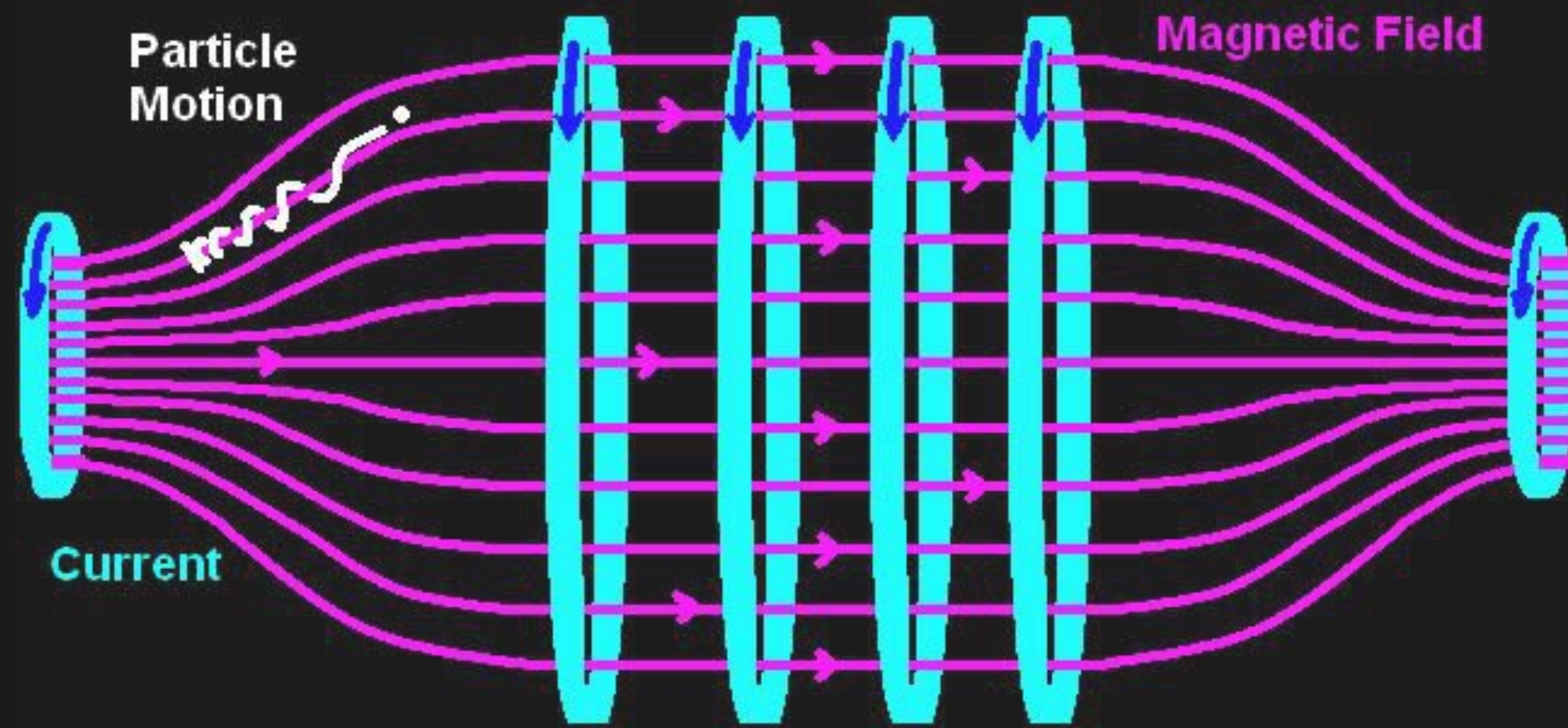
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Conservation of magnetic moment:

$$\mu = \frac{W_{\perp}}{B} \rightarrow W_{\parallel}$$

conservation of energy

# Mirror machines: old fusion concept, basis is conservation of $\mu$



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Conservation of magnetic moment:

$$\mu = \frac{W_{\perp}}{B}$$

The equation is annotated with a blue arrow pointing up next to  $W_{\perp}$  and a red arrow pointing up next to  $B$ .

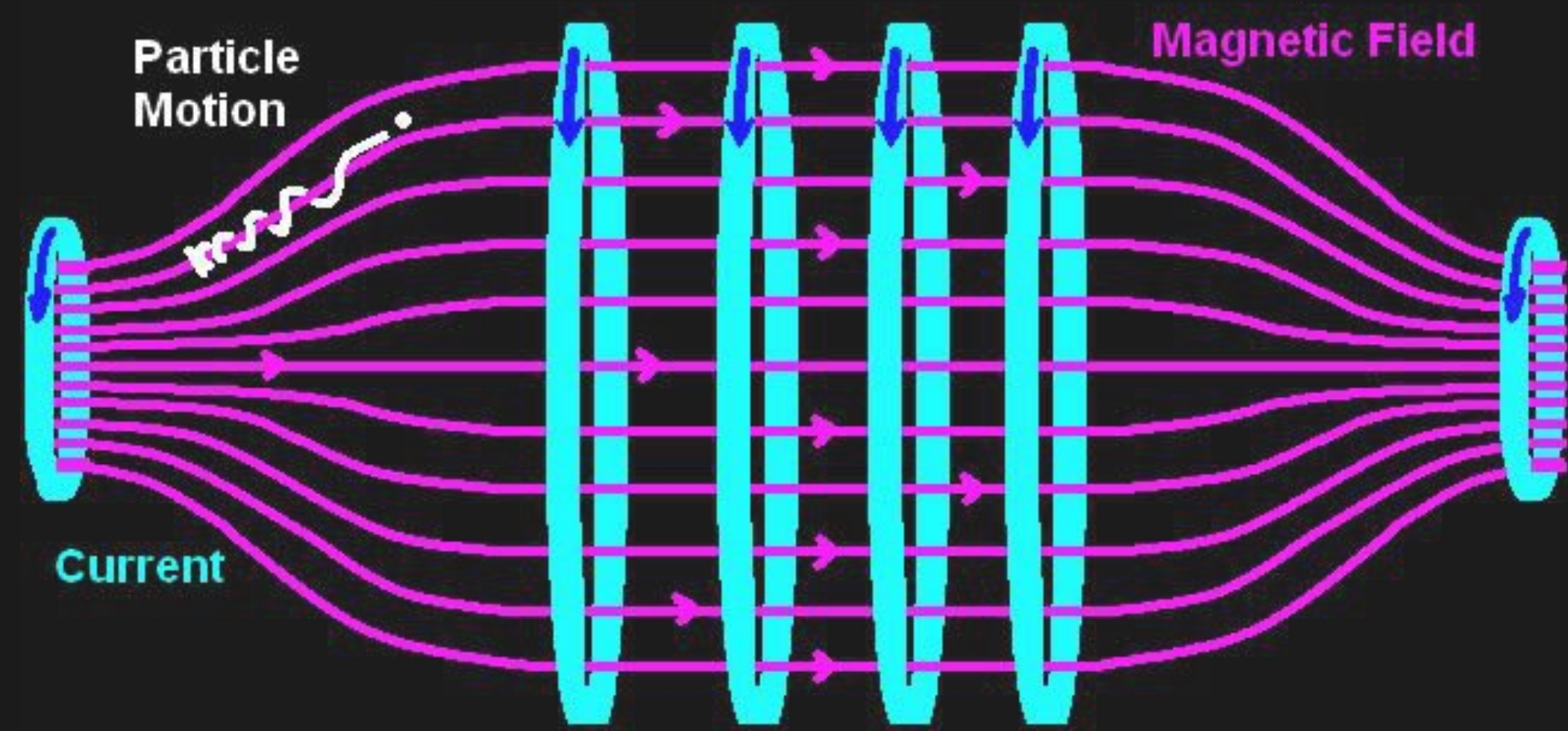
conservation of  
energy

$$W_{\parallel}$$

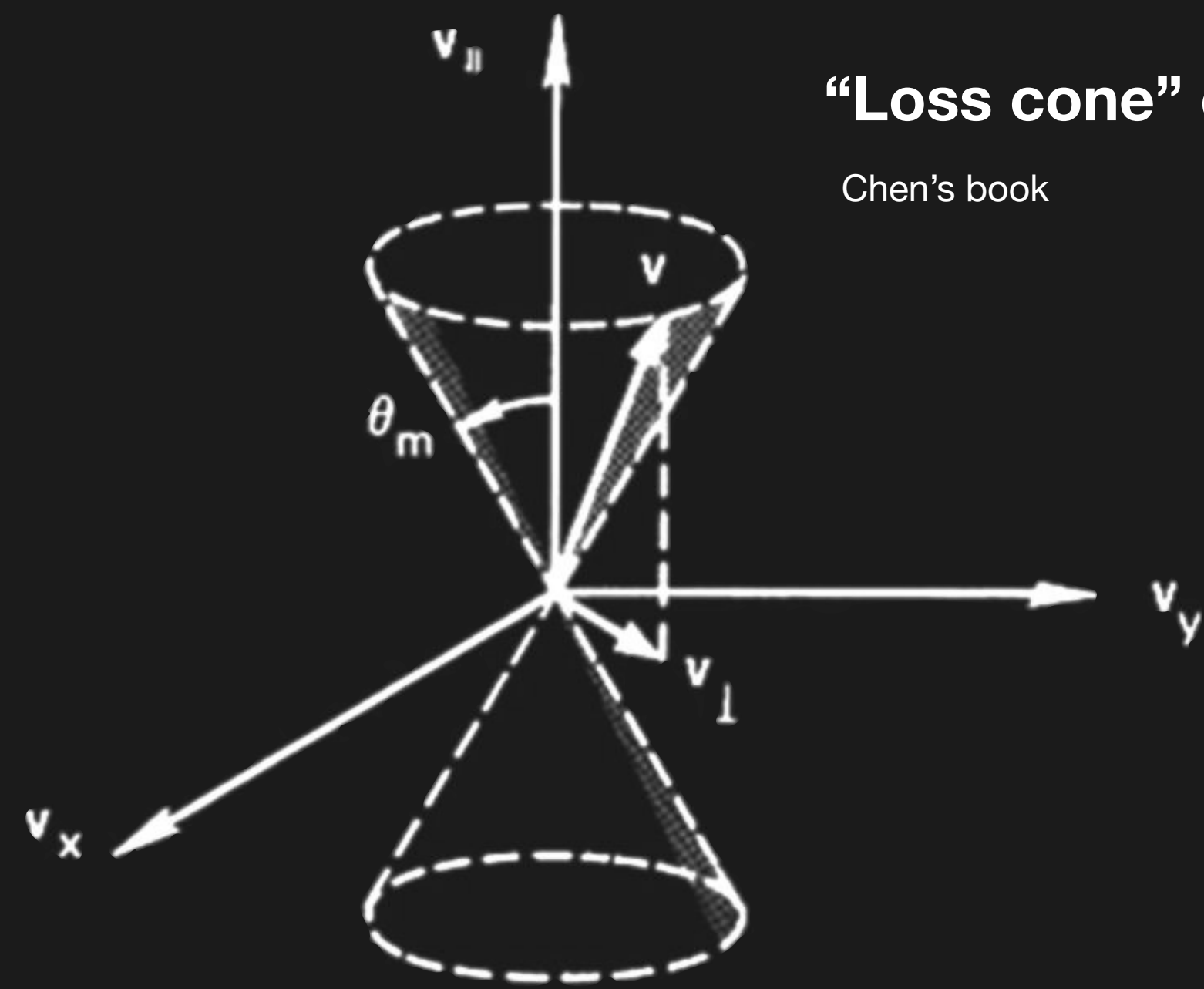
The equation is annotated with a blue arrow pointing down next to  $W_{\parallel}$ .

particle reflected

# Mirror machines: old fusion concept, basis is conservation of $\mu$



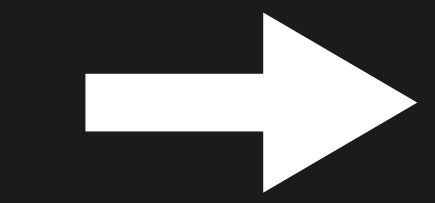
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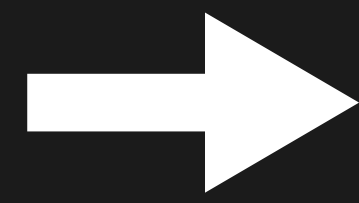
“Loss cone” distribution

Conservation of magnetic moment:

$$\mu = \frac{W_{\perp}}{B}$$



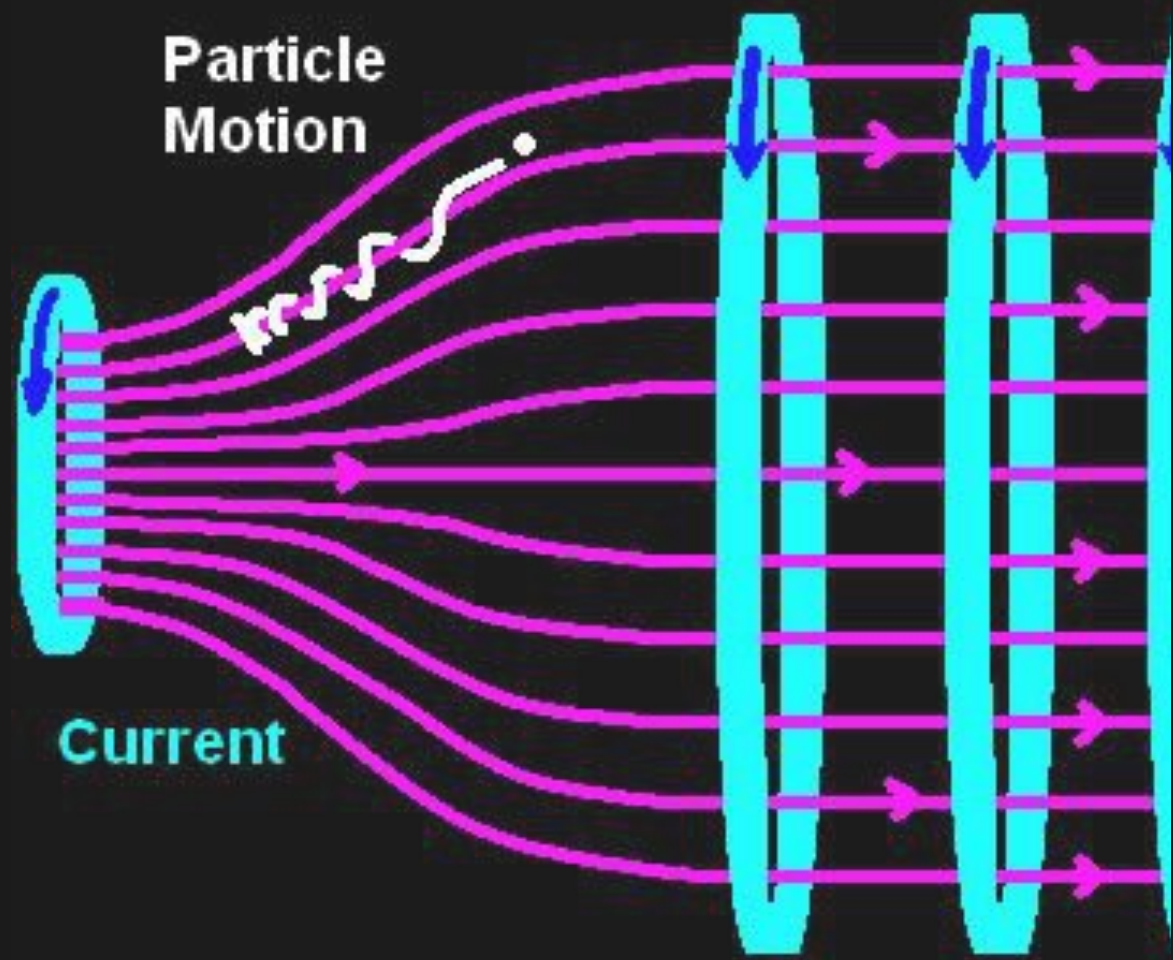
$$W_{\parallel}$$



particle reflected

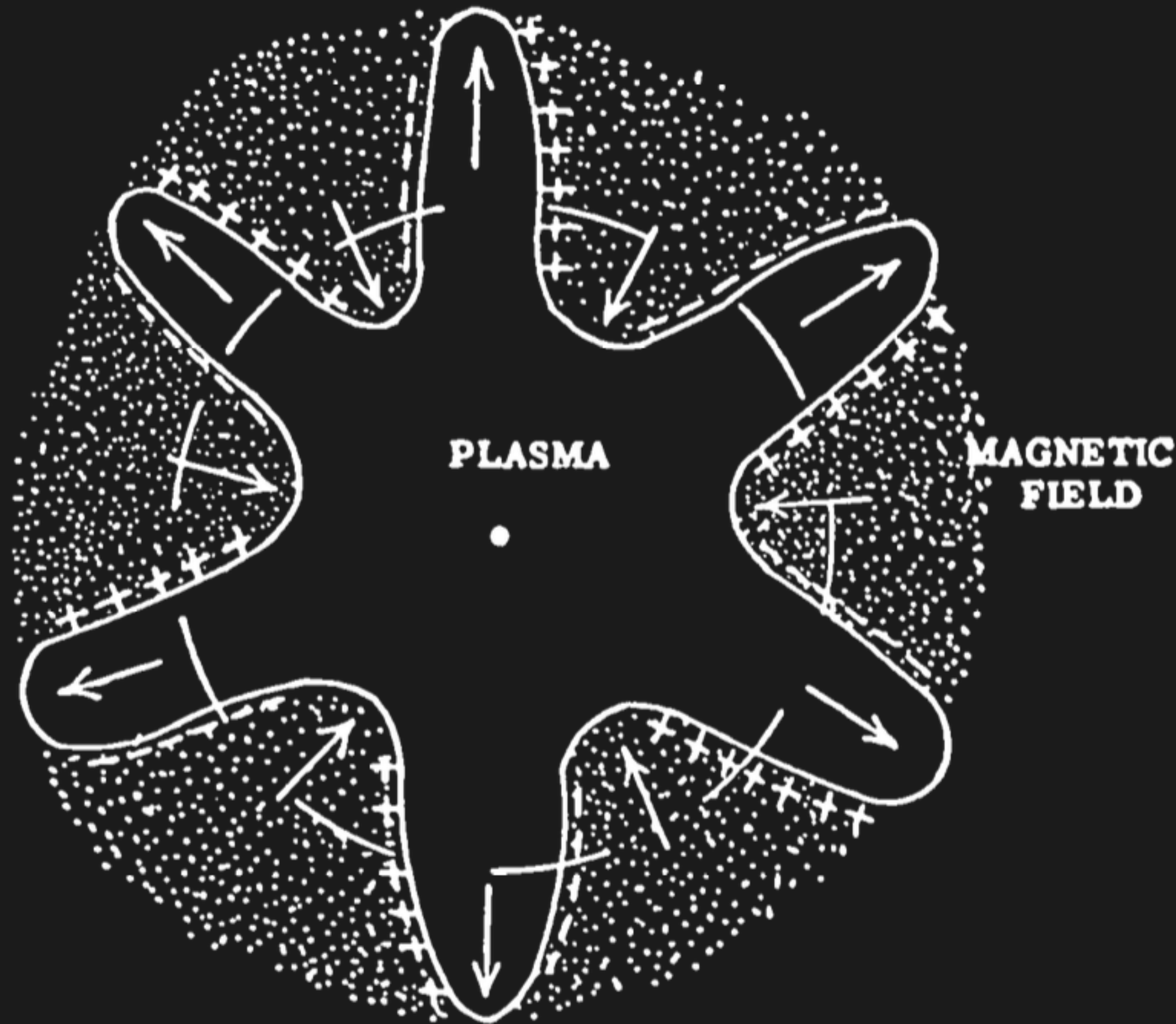
conservation of energy

# Mirror machines: old of $\mu$



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Conservation of magnetic moment



“Loss cone” distribution

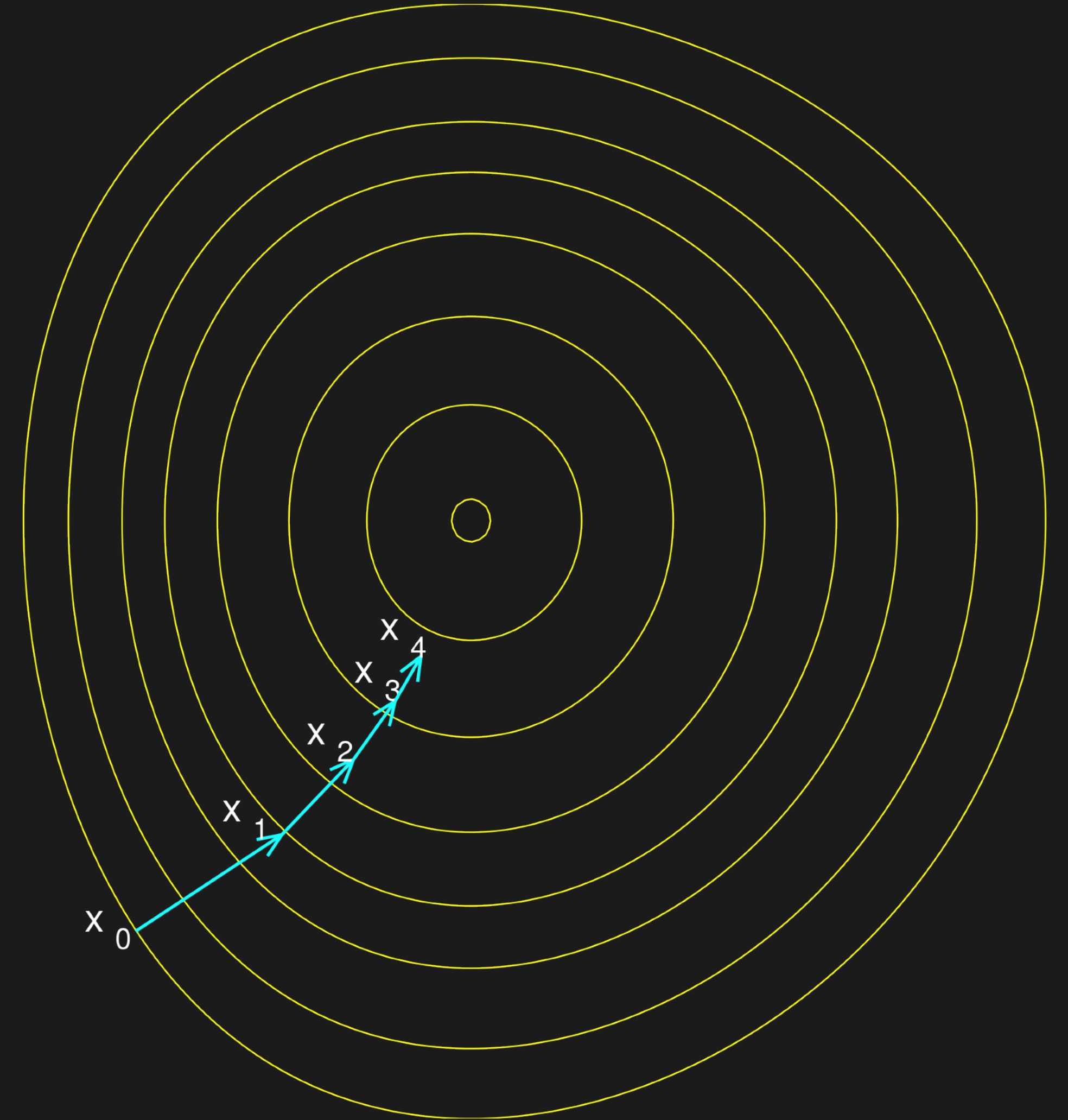
Chen’s book



particle reflected

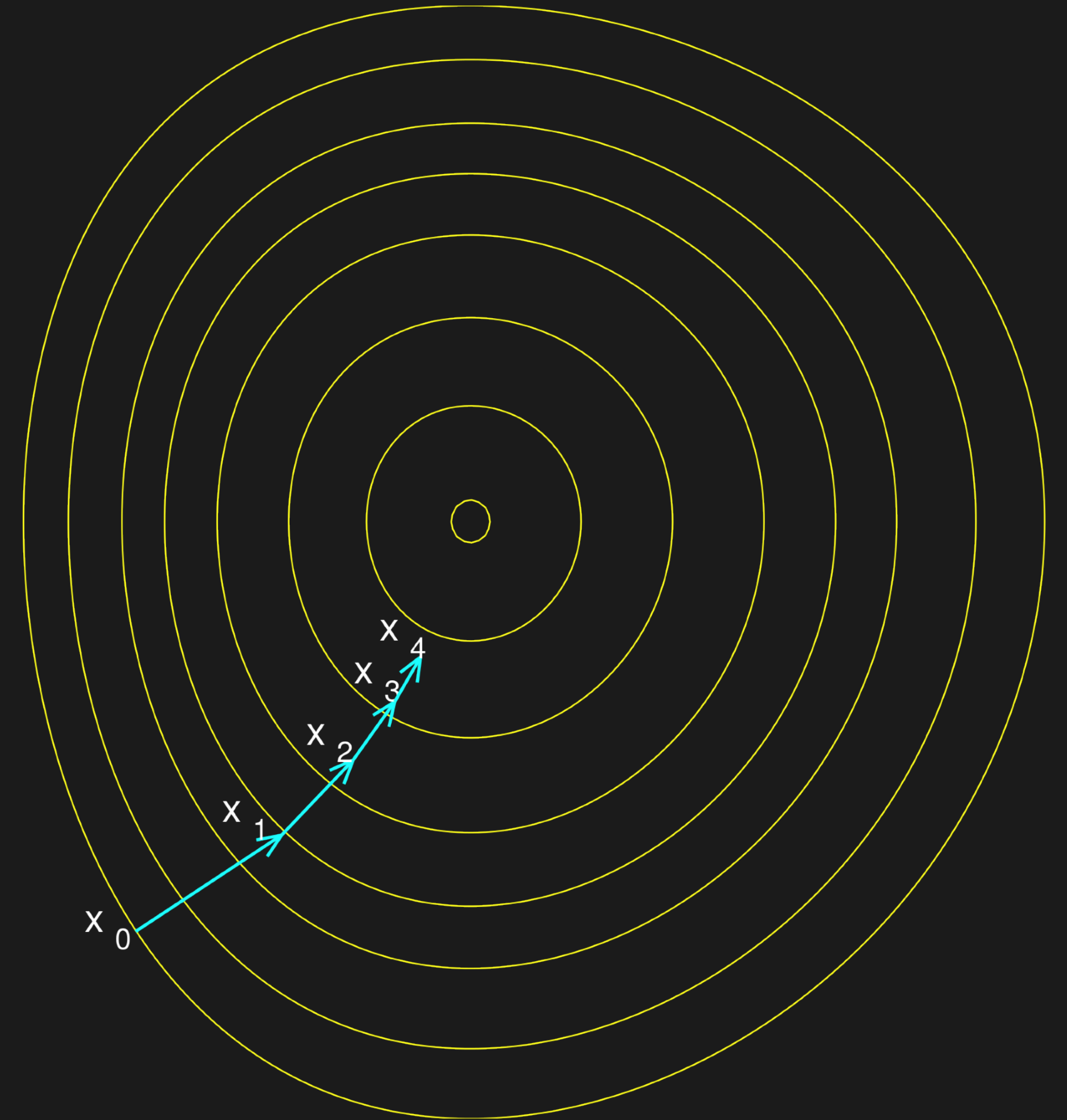
FIG. 30. Schematic illustration of the high- $m$  flute instability in a simple mirror field, showing polarization fields and directions of unstable motion.

# Optimization using gradient descent



# Optimization using gradient descent

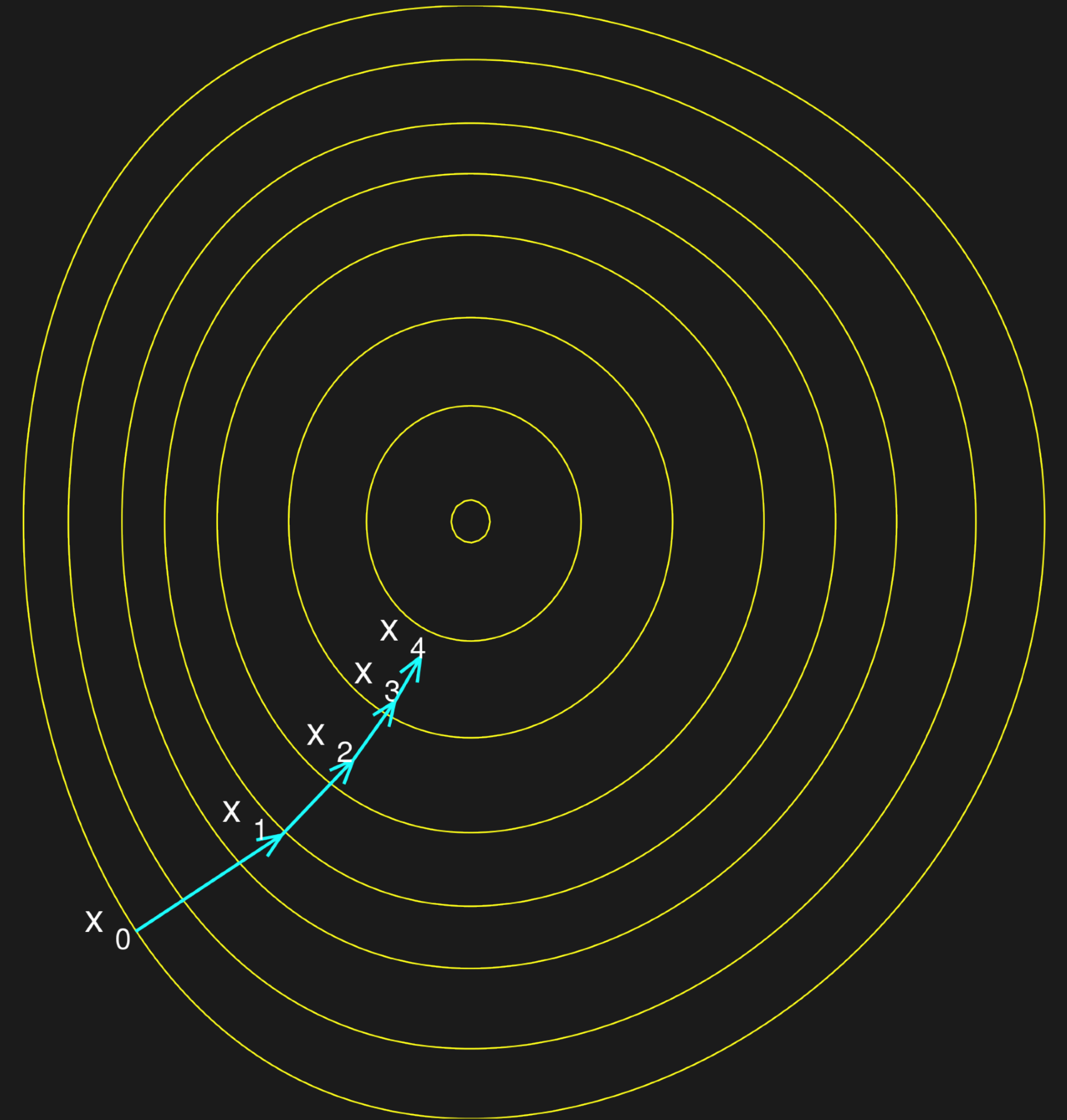
- Update values based on gradient
  - $\vec{x} := \vec{x} - \nabla_{\vec{x}} \mathcal{C} \cdot \lambda$
- $\mathcal{C}$  is the cost function (quantity to minimize)
- $\lambda$  is step size (chosen by trial-and-error in this case)



# Optimization using gradient descent



you get there in the end

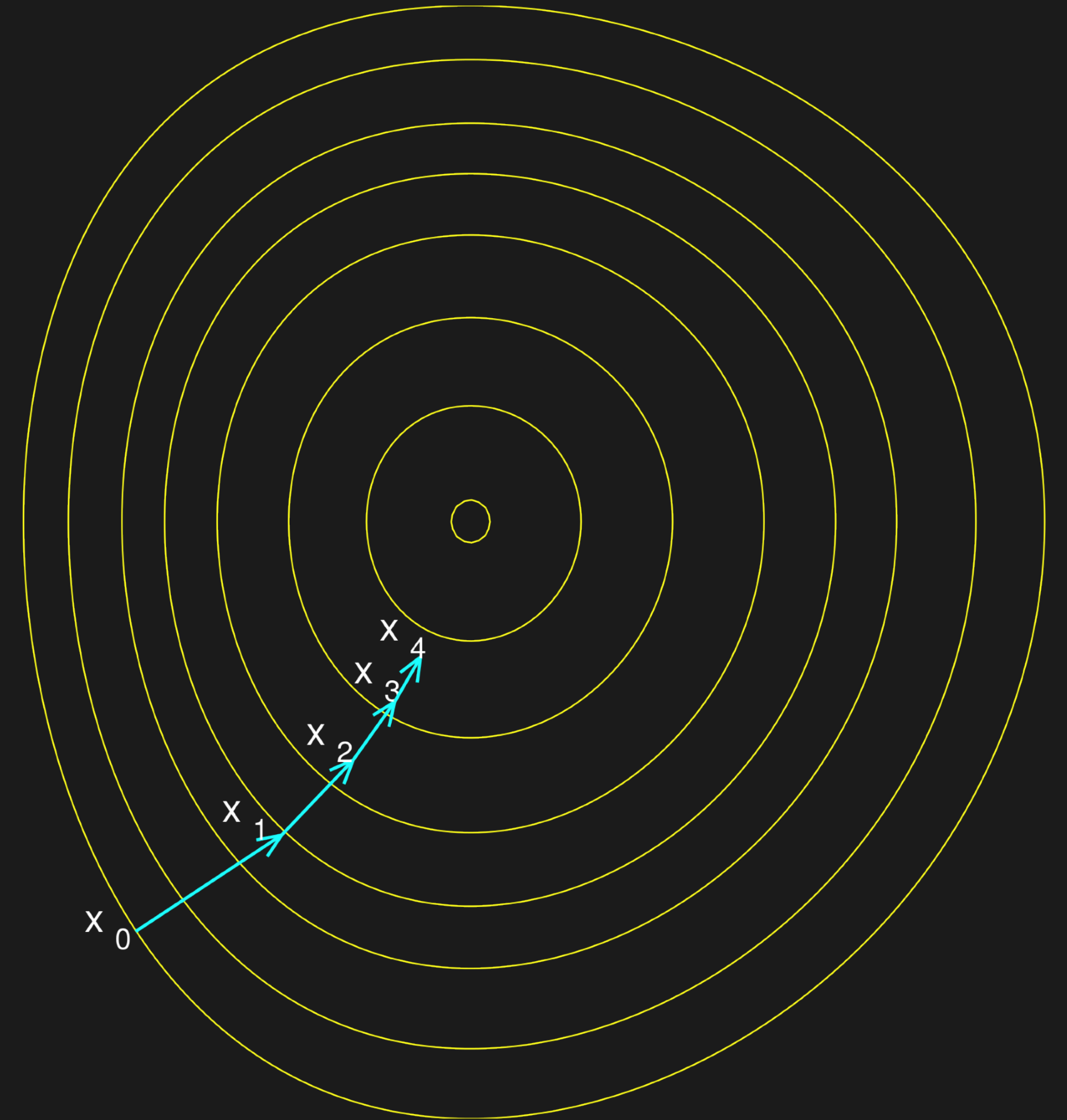


# Optimization using gradient descent

- Update values based on gradient

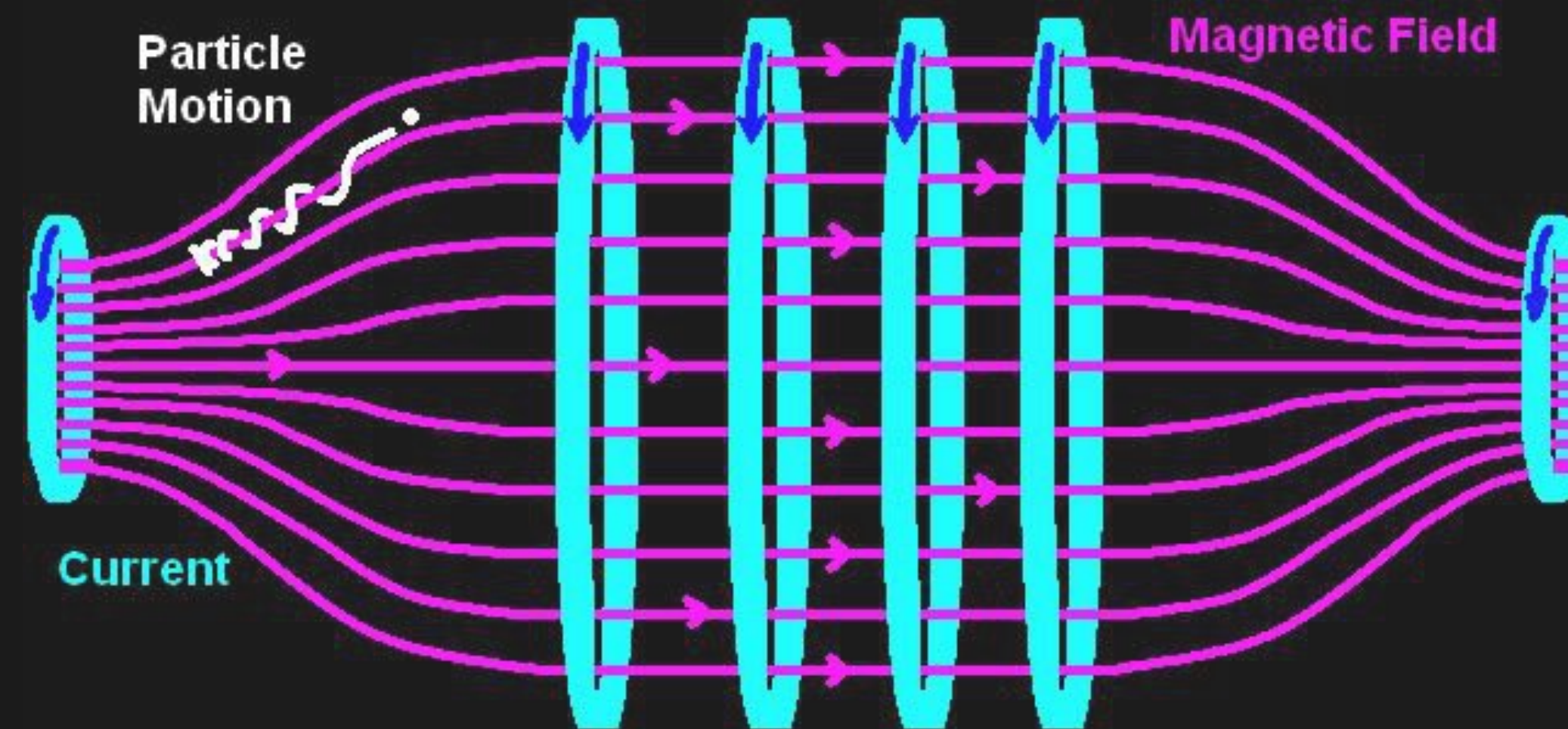


you get there in the end



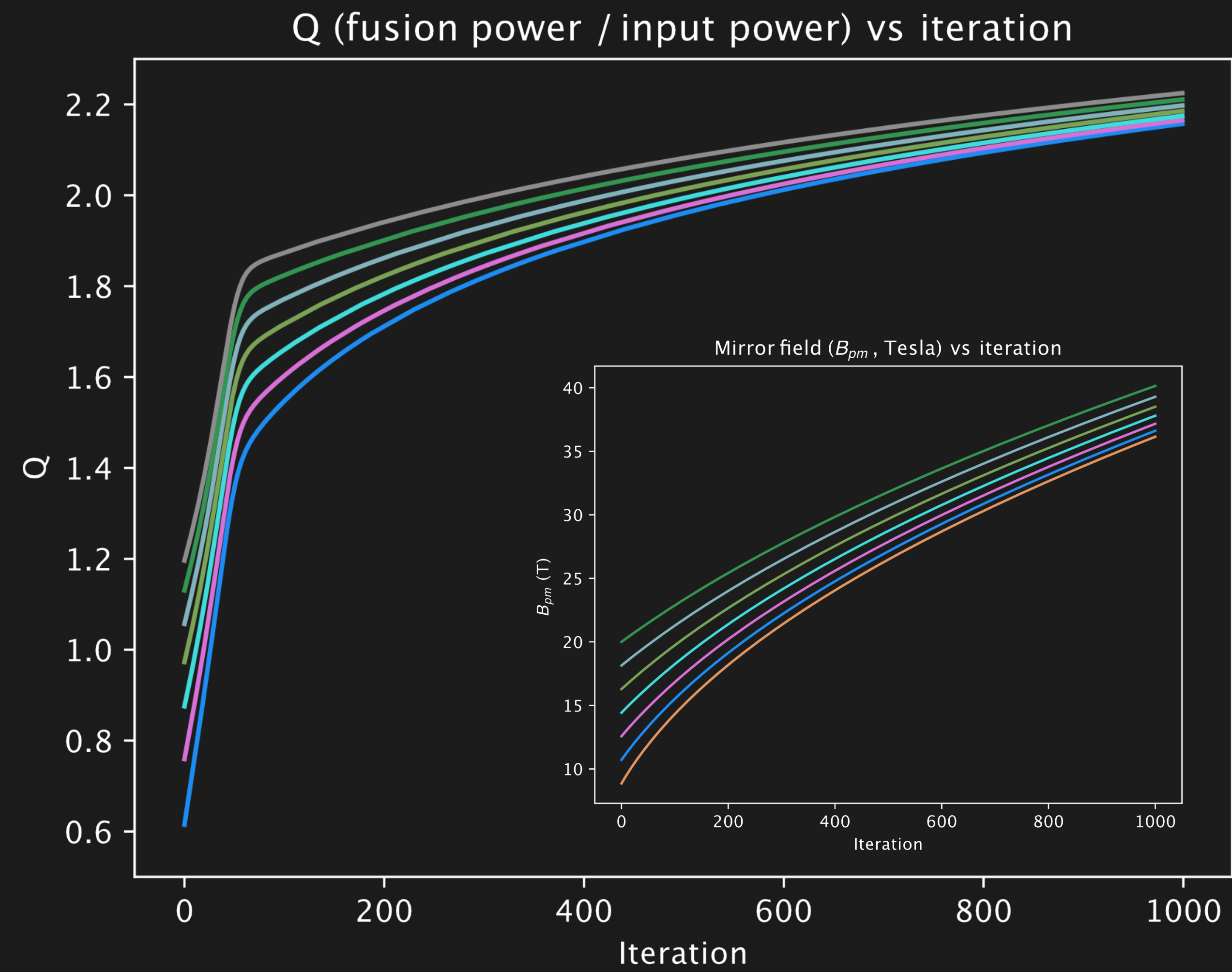
so lets optimize a simple mirror

so lets optimize a simple mirror

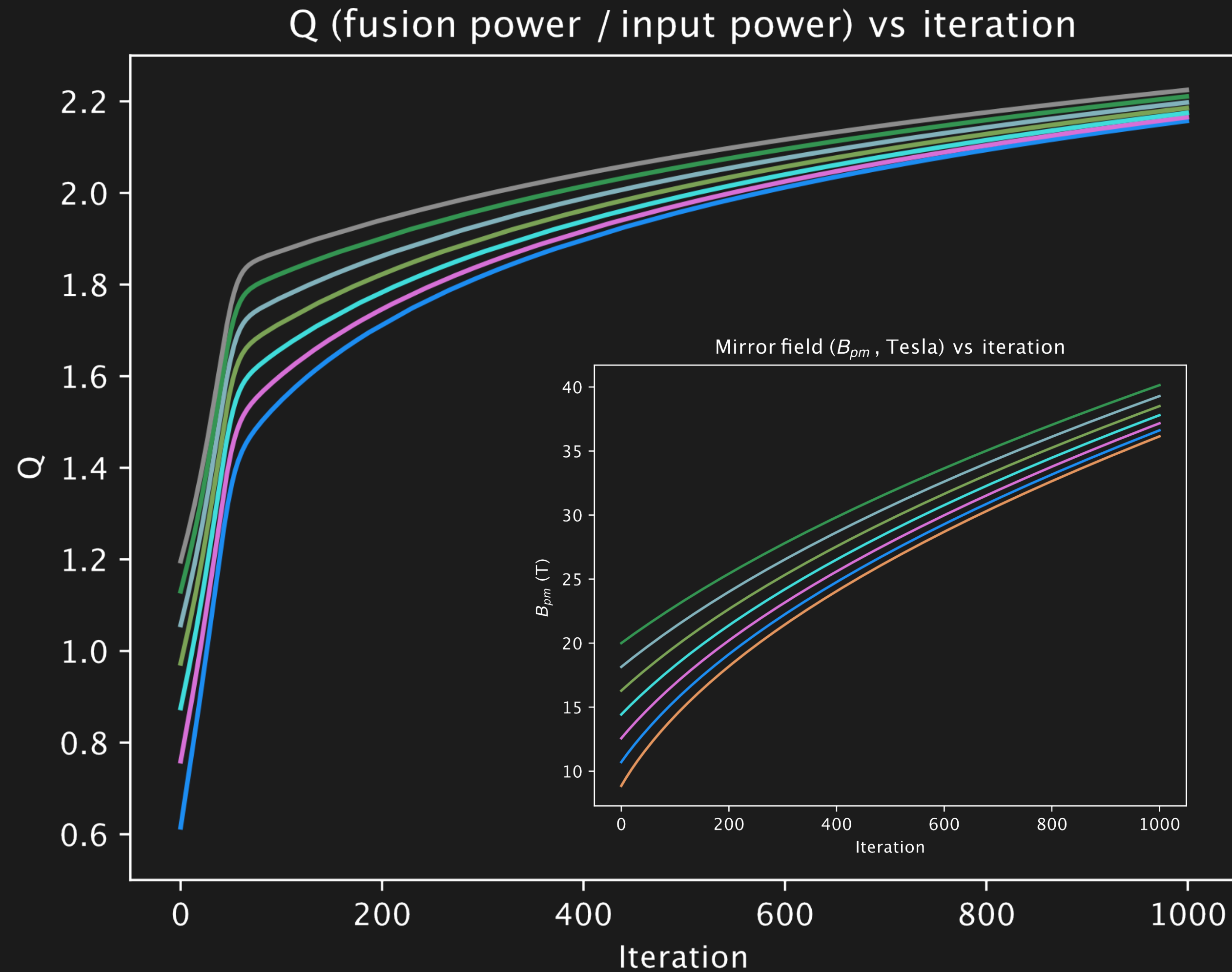


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# Fun fact: simple mirrors kinda aren't great

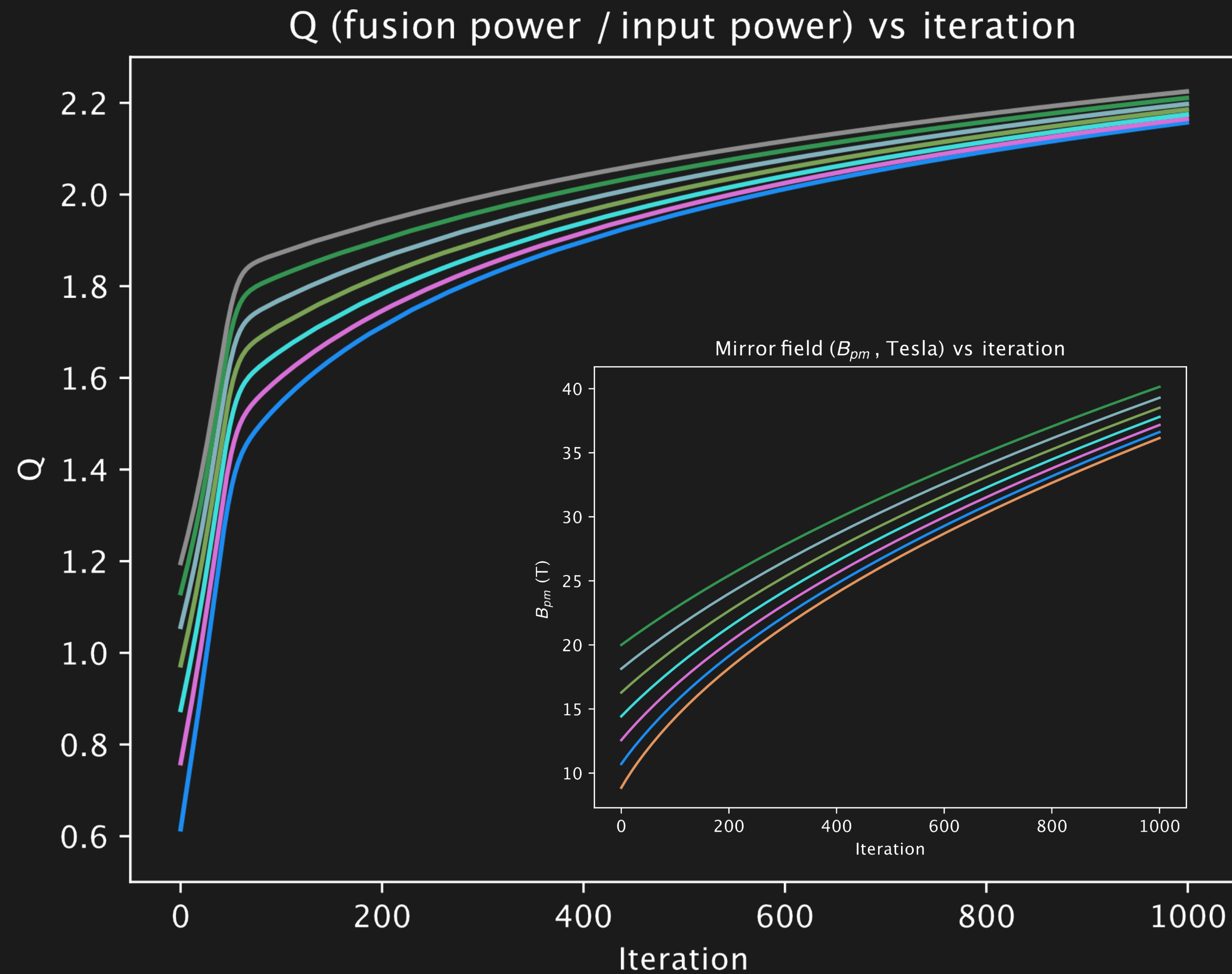


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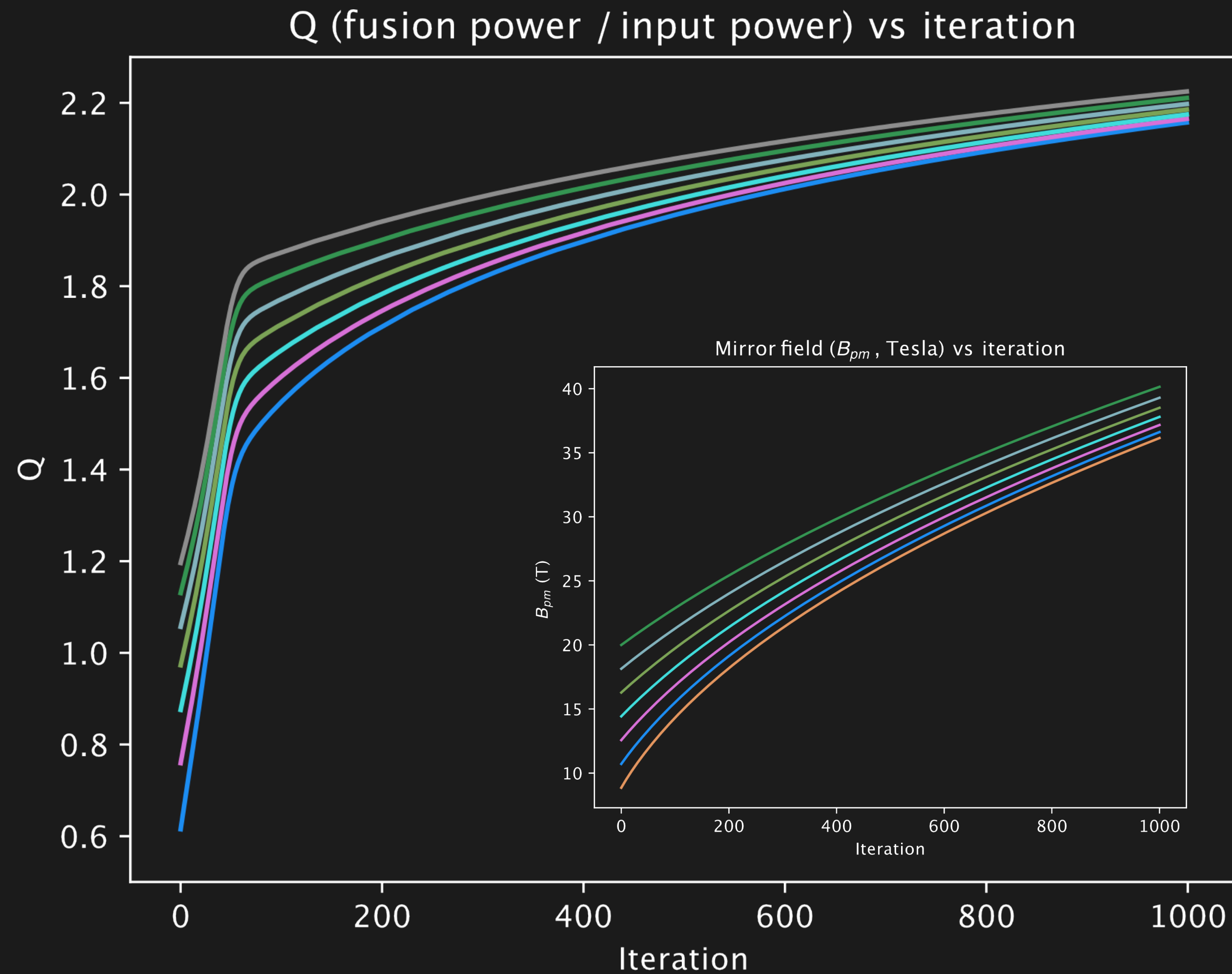
- Mirror field optimizes to 35+ T

# Fun fact: simple mirrors kinda aren't great



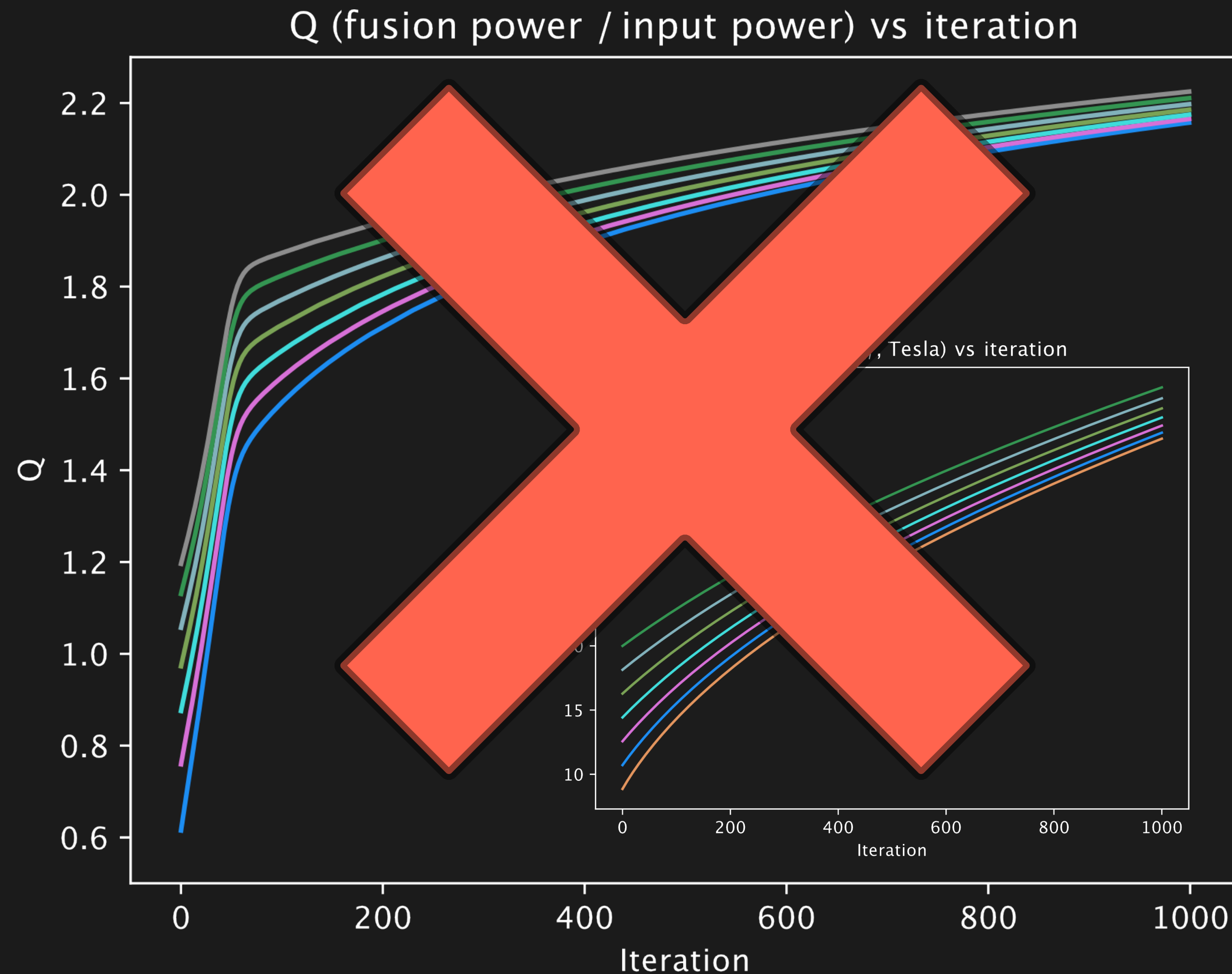
- Mirror field optimizes to 35+ T
- Q optimizes to a little above 2

# Fun fact: simple mirrors kinda aren't great



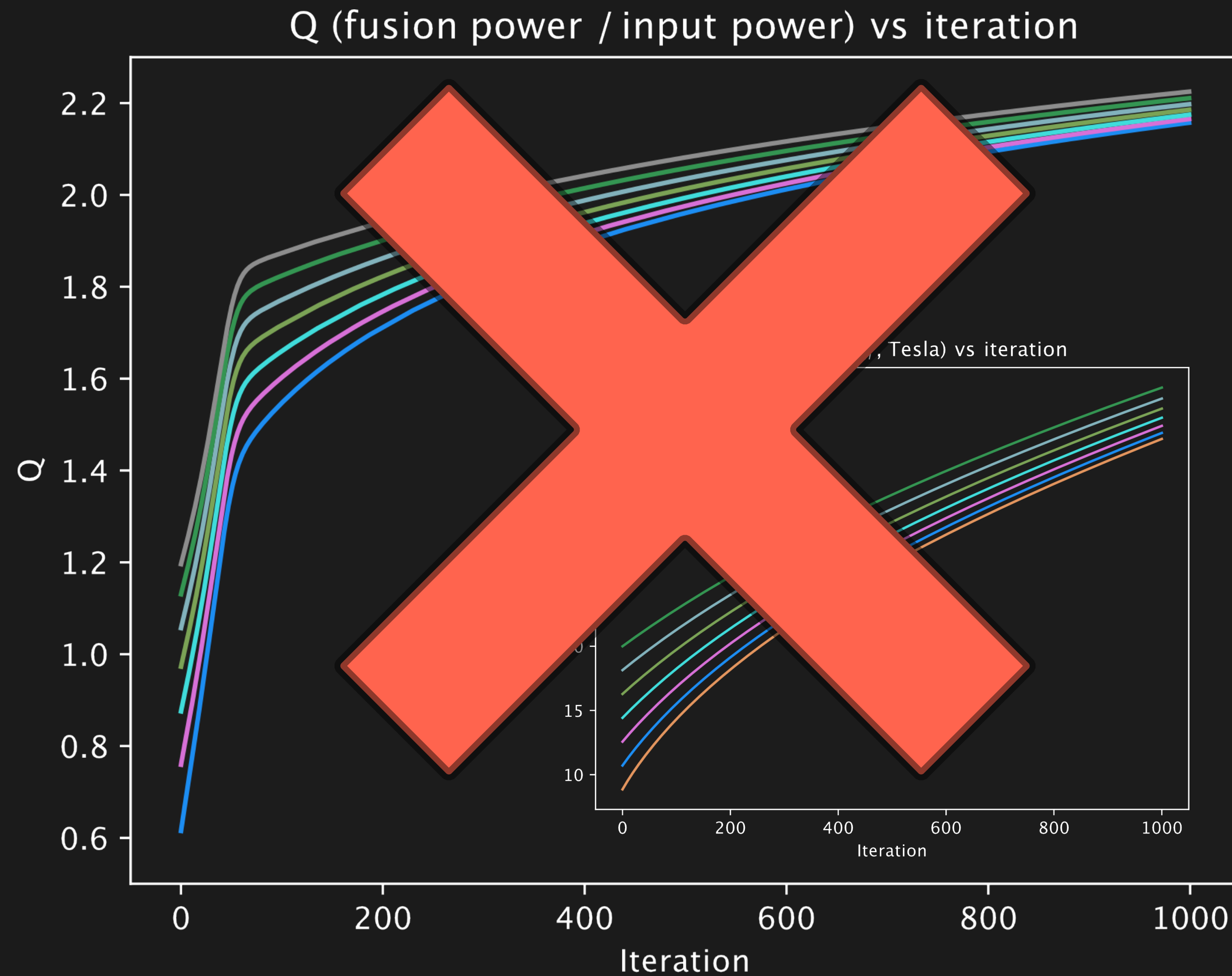
- Mirror field optimizes to 35+ T
- $Q$  optimizes to a little above 2
- 1 MeV NBI

# Fun fact: simple mirrors kinda aren't great



- Mirror field optimizes to 35+ T
- Q optimizes to a little above 2
- 1 MeV NBI
- Simple mirrors aint gonna work \*

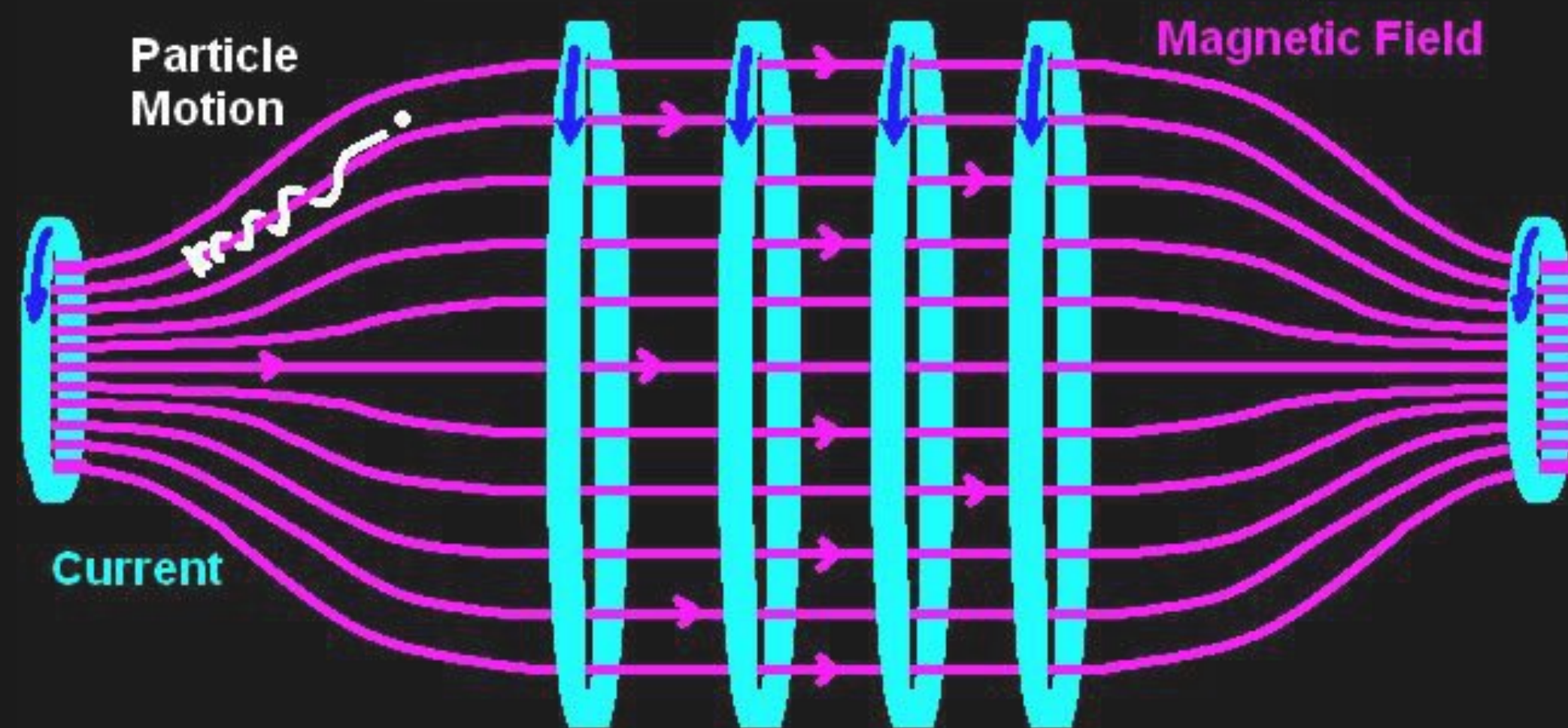
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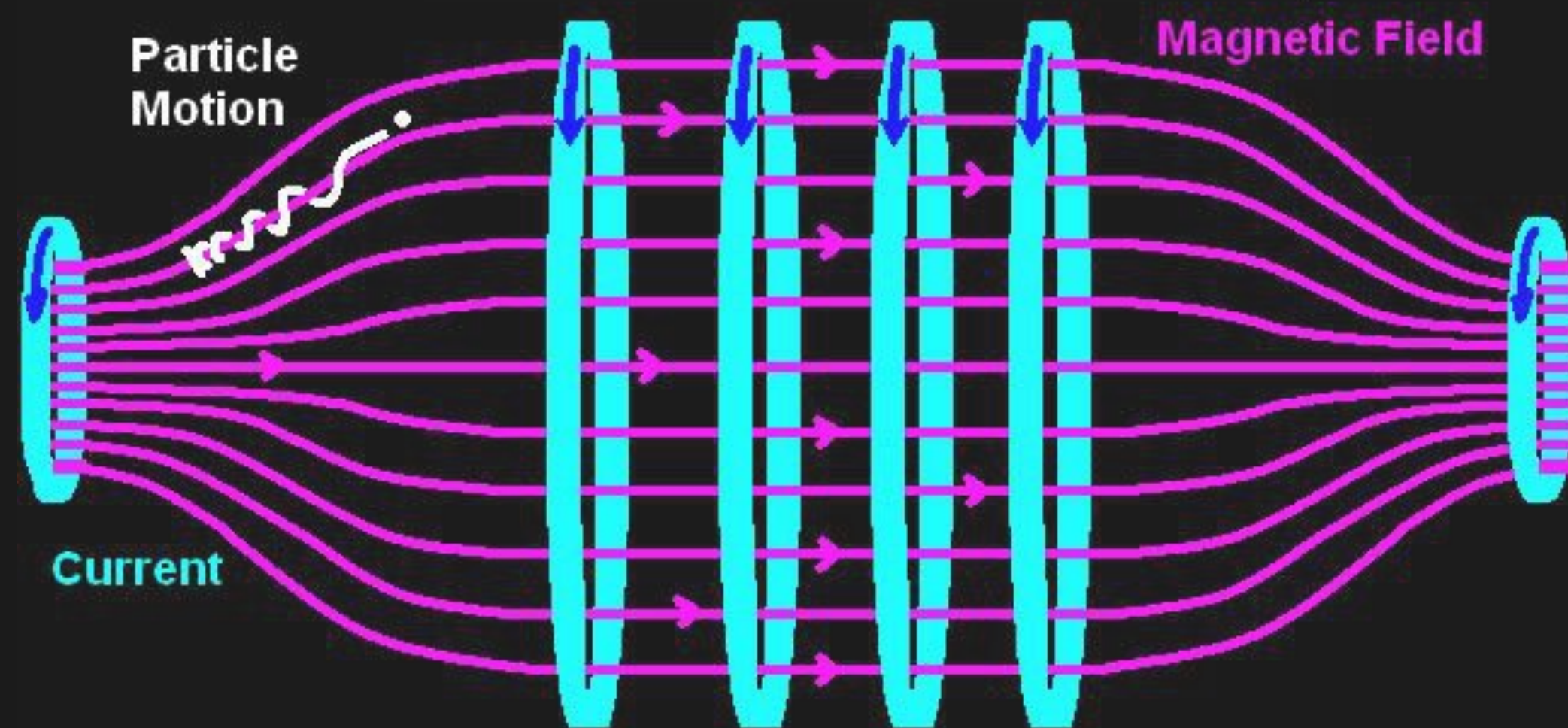
- Mirror field optimizes to 35+ T
- Q optimizes to a little above 2
- 1 MeV NBI
- Simple mirrors aint gonna work \*

\* welllllll maybe we can use a gas-dynamic trap for a neutron source





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so add more mirrors

“tandem mirror”



(okay it's actually 3 but you get the point)

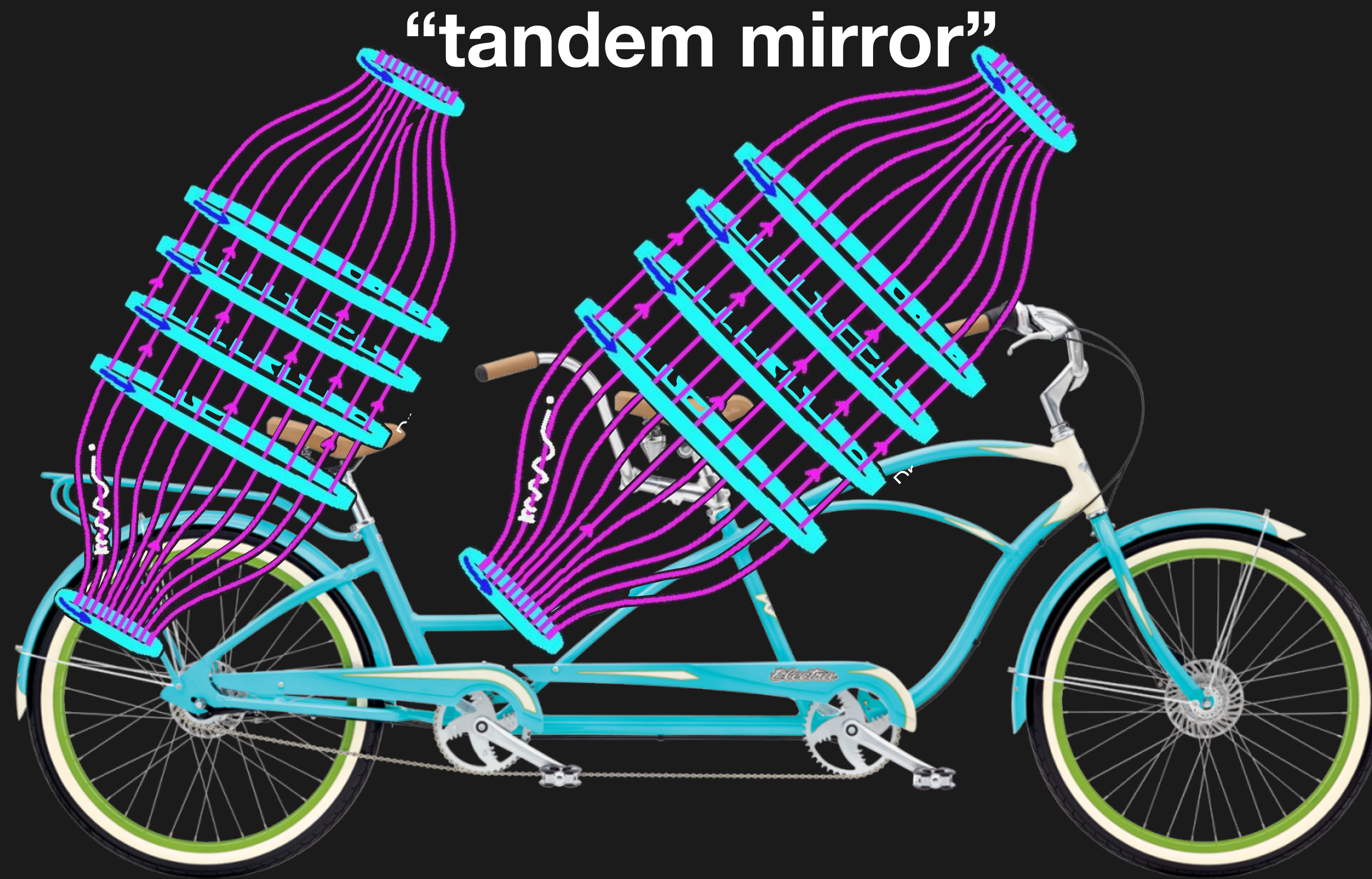
so add more mirrors

“tandem mirror”



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so add more mirrors



(okay it's actually 3 but you get the point)

so add more mirrors



so add more mirrors



*Fig 1. Toast sandwich / tandem mirror*

so add more mirrors



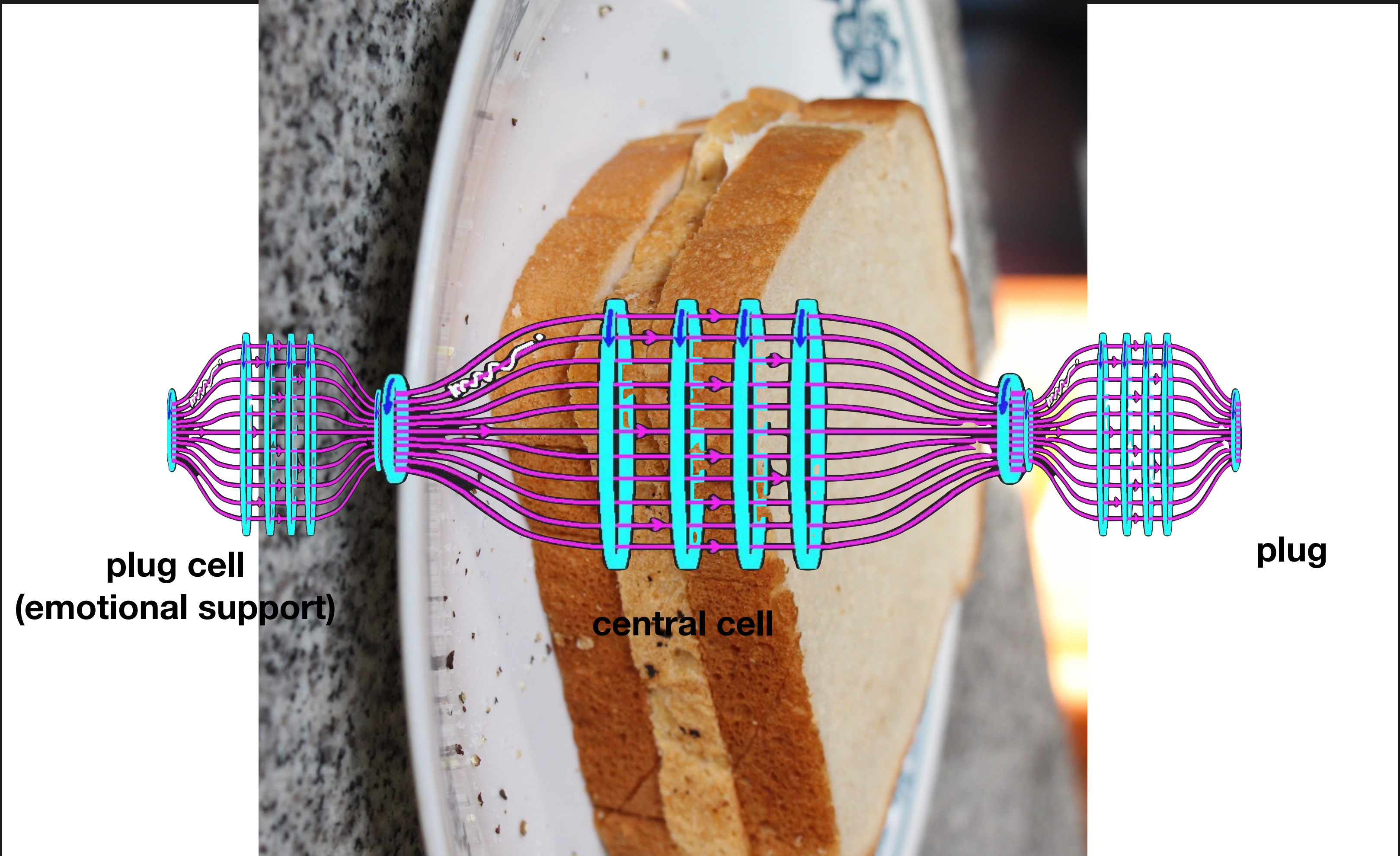
*Fig 1. Toast sandwich / tandem mirror*

so add more mirrors



*Fig 1. Toast sandwich / tandem mirror*

so add more mirrors



*Fig 1. Toast sandwich / tandem mirror*

# Let's optimize that tandem

**It does okay**

# Let’s optimize that tandem

## Cost function:

```
(9.42477796076938e-17*B_cc*B_p**2*B_pm*L_cc*Te*beta*n_ccr*r_b**2*(1 - Te/511)/(4.02670907669414e-22*Te + 4.02670907669414e-22*Ti) +
795774715.026276*B_cc*L_cc*cost_HTS_kAm*(6.28318530717959*a_wall_r*r_b*sqrt(B_pm/B_cc) + 6.28318530717959*d_blanket + 6.28318530717959*d_vv)/
coil_spacing_cc + 32995390.6601843*pi*B_p**3*B_pm*L_p*sqrt(Te)*Z_eff*beta**2*r_b**2/(Te + Ti)**2 + 3.0e-17*pi*B_p**3*B_pm*L_p*Te*beta*r_b**2*(1 - Te/511)/
(4.02670907669414e-22*Te + 4.02670907669414e-22*Ti) + 8.01088317e-16*pi*B_p*B_pm*Eb*L_p*beta*r_b**2*(4.16666666666667e-17*B_p**2*beta*log(10)/
(Eb**1.5*(4.02670907669414e-22*Te + 4.02670907669414e-22*Ti)*log(B_pm/(B_p*sqrt(1 - beta)))) + 0.296691351241568*Te*sqrt(Ti)/(B_p*B_pm*r_b**2))/
(4.02670907669414e-22*Te + 4.02670907669414e-22*Ti) + 1591549430.05255*B_pm*cost_HTS_kAm*(a_wall_r*r_b + d_blanket +
d_vv)*(6.28318530717959*a_wall_r*r_b + 6.28318530717959*d_blanket + 6.28318530717959*d_vv) +
1591549430.05255*B_pm*cost_HTS_kAm*(a_wall_r*r_b*sqrt(B_pm/B_p) + d_blanket + d_vv)*(6.28318530717959*a_wall_r*r_b*sqrt(B_pm/B_p) +
6.28318530717959*d_blanket + 6.28318530717959*d_vv) + 103658076.90036*B_p**4*B_pm*L_cc*sqrt(Te)*Z_eff*beta**2*n_ccr**2*r_b**2/(B_cc*(Te + Ti)**2) +
7.55007951469143e-15*B_p**2*B_pm*L_cc*beta*n_ccr*r_b**2*(Te*Te_ccr + Te*T_i_ccr)*(8.0e-17*B_p**2*Z_eff**4*beta*n_ccr/
(sqrt(mu)*(Te*T_i_ccr)**1.5*(4.02670907669414e-22*Te + 4.02670907669414e-22*T_i)*(1/n_ccr)**(1/T_i_ccr)*log(1/n_ccr)*log(1 + 2*B_pm/(B_cc*sqrt(1 - Piecewise((0.9,
4.0267e-22*B_p**2*beta*n_ccr*(Te*Te_ccr + Te*T_i_ccr))/(B_cc**2*(4.02670907669414e-22*Te + 4.02670907669414e-22*T_i)) > 0.9),
(4.0267e-22*B_p**2*beta*n_ccr*(Te*Te_ccr + Te*T_i_ccr))/(B_cc**2*(4.02670907669414e-22*Te + 4.02670907669414e-22*T_i)), True)))))) +
0.296691351241568*Te*Te_ccr*sqrt(Te*T_i_ccr)/(B_cc*B_pm*r_b**2))/(B_cc*(4.02670907669414e-22*Te + 4.02670907669414e-22*T_i))/
(-8.421040388304e-17*pi*B_p*B_pm*Eb*Ef_NBI*L_p*beta*r_b**2*(4.16666666666667e-17*B_p**2*beta*log(10)/(Eb**1.5*(4.02670907669414e-22*Te +
4.02670907669414e-22*T_i)*log(B_pm/(B_p*sqrt(1 - beta)))) + 0.296691351241568*Te*sqrt(Ti)/(B_p*B_pm*r_b**2))/(4.02670907669414e-22*Te +
4.02670907669414e-22*T_i) + 262800*Ef_DEC*(1.72921317451384e+24*pi*B_p**3*B_pm*DD_reac*L_p*beta**2*r_b**2*(1 - T_frac)**2/(Te + Ti)**2 +
6.91684313304943e+24*pi*B_p**3*B_pm*DT_reac*L_p*T_frac*beta**2*r_b**2*(1 - T_frac)/(Te + Ti)**2 + 82343.3706315408*pi*B_p**3*B_pm*L_p*beta**2*r_b**2*(Eb -
Te)*log(10)/(Eb**1.5*(Te + Ti)**2*log(B_pm/(B_p*sqrt(1 - beta)))) + 2.71624170277168e+24*B_p**4*B_pm*DD_reac*L_cc*beta**2*n_ccr**2*r_b**2*(1 - T_frac)**2/(B_cc*(Te
+ Ti)**2) + 1.08649517864105e+25*B_p**4*B_pm*DT_reac*L_cc*T_frac*beta**2*n_ccr**2*r_b**2*(1 - T_frac)/(B_cc*(Te + Ti)**2) +
6.04006361175315e-38*B_p**4*B_pm*L_cc*Z_eff**4*beta**2*n_ccr**2*r_b**2*(6.16736274022137e+42/(Te + Ti)**2)*(Te*Te_ccr + Te*T_i_ccr)/
(B_cc*sqrt(mu)*(Te*T_i_ccr)**1.5*(1/n_ccr)**(1/T_i_ccr)*log(1/n_ccr)*log(1 + 2*B_pm/(B_cc*sqrt(1 - Piecewise((0.9, 4.0267e-22*B_p**2*beta*n_ccr*(Te*Te_ccr + Te*T_i_ccr)/
(B_cc**2*(4.02670907669414e-22*Te + 4.02670907669414e-22*T_i)) > 0.9), (4.0267e-22*B_p**2*beta*n_ccr*(Te*Te_ccr + Te*T_i_ccr)/(B_cc**2*(4.02670907669414e-22*Te
+ 4.02670907669414e-22*T_i)), True)))))) - 262800*Ef_ECH*(9.42477796076938e-24*B_cc*B_p**2*B_pm*L_cc*Te*beta*n_ccr*r_b**2*(1 - Te/511)/
(4.02670907669414e-22*Te + 4.02670907669414e-22*T_i) + 6.59907813203687*pi*B_p**3*B_pm*L_p*sqrt(Te)*Z_eff*beta**2*r_b**2/(Te + Ti)**2 +
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10.3665807690036*B_p**4*B_pm*L_cc*sqrt(Te)*Z_eff*beta**2*n_ccr**2*r_b**2/(B_cc*(Te + Ti)**2) +
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(sqrt(mu)*(Te*T_i_ccr)**1.5*(4.02670907669414e-22*Te + 4.02670907669414e-22*T_i)*(1/n_ccr)**(1/T_i_ccr)*log(1/n_ccr)*log(1 + 2*B_pm/(B_cc*sqrt(1 - Piecewise((0.9,
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0.296691351241568*Te*Te_ccr*sqrt(Te*T_i_ccr)/(B_cc*B_pm*r_b**2))/(B_cc*(4.02670907669414e-22*Te + 4.02670907669414e-22*T_i)) +
262800*Ef_TE*(8.176696703712e+24*pi*B_p**3*B_pm*DD_reac*L_p*beta**2*r_b**2*(1 - T_frac)**2/(Te + Ti)**2 +
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+ 4.02670907669414e-22*T_i)), True)))))) + 1.28439251475068e+25*B_p**4*B_pm*DD_reac*L_cc*beta**2*n_ccr**2*r_b**2*(1 - T_frac)**2/(B_cc*(Te + Ti)**2) +
4.37702343395396e+25*B_p**4*B_pm*DT_reac*L_cc*T_frac*beta**2*n_ccr**2*r_b**2*(1 - T_frac)/(B_cc*(Te + Ti)**2)))
```

It does okay

# Let's optimize that tandem

- Cost function

$$\mathcal{C} = \frac{\text{Magnet cost} + \text{heating cost}}{\text{Electric energy produced}}$$

- No blanket / structure costs (yet)

**It does okay**

# Future work: a one-man differentiable reactor design study. haha, jk... unless...?

- Near term
  - Add sanity checks (e.g., magnet structure loads, NBI shine-through)
  - Improve mirror cell particle + energy balance
  - Improve plug confinement time estimates
  - Include instability mitigation in optimization
    - DCLC, interchange (pretty much solved)
  - Incorporate better radial transport model (still an open question)
  - Implement thermal barrier tandem mirrors (enhances axial confinement)
  - Do a bunch of parameter scans (over things like  $Z_{\text{eff}}$ )
- Long term: automating reactor design

thanks

- bottom text



not my dog but she's hella cute